Mathematical Logics
Propositional Logic *

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Reference(s):

- **Francesco Berto, Logica da zero a Gödel, Laterza, 2018 (capitolo 1)**

**Originally by Luciano Serafini and Chiara Ghidini*
Modified by Fausto Giunchiglia and Mattia Fumagalli

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Logical consequence

Definition (Logical consequence)

A formula A is a logical consequence of a set of formulas Γ, in symbols

$$\Gamma \models A$$

Iff for any interpretation I that satisfies all the formulas in Γ, I satisfies A

Example (Logical consequence)

- $p \models p \vee q$
- $q \vee p \models p \vee q$
- $p \vee q, p \rightarrow r, q \rightarrow r \models r$
- $p \rightarrow q, p \models q$
- $p, \neg p \models q$

Proving Logical consequence using the truth tables

Use the truth tables method to determine whether $p \land \neg q \rightarrow p \land q$ is a logical consequence of $\neg p$.

p	q	$\neg p$	$p \land \neg q$	$p \land q$	$p \land \neg q \rightarrow p \land q$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	F	F	T

Logical Equivalence

Definition (Logical equivalence)

Two formulas F and G are logically equivalent (denoted with $F \equiv G$) if for each interpretation I,

$$I(F) = I(G).$$

Truth Tables: Example (5)

Use the truth tables method to determine whether $p \rightarrow (q \wedge \neg q)$ and $\neg p$ are logically equivalent.

p	q	$q \wedge \neg q$	$p \rightarrow (q \wedge \neg q)$	$\neg p$
T	T	F	F	F
T	F	F	F	F
F	T	F	T	T
F	F	F	T	T

Properties of propositional logical consequence

Proposition

If Γ and Σ are two sets of propositional formulas and A and B two formulas, then the following properties hold:

Reflexivity $\{A\} \models A$

Monotonicity If $\Gamma \models A$ then $\Gamma \cup \Sigma \models A$

Cut If $\Gamma \models A$ and $\Sigma \cup \{A\} \models B$ then $\Gamma \cup \Sigma \models B$

Compactness If $\Gamma \models A$, then there is a finite subset $\Gamma_0 \subseteq \Gamma$, such that $\Gamma_0 \models A$

Deduction theorem If $\Gamma, A \models B$ then $\Gamma \models A \rightarrow B$

Refutation principle $\Gamma \models A$ iff $\Gamma \cup \{\neg A\}$ is unsatisfiable

NOTE: *vice versa of deduction theorem trivial*

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